Spontaneous Symmetry Breaking Mechanism in Light-Front Quantized Field Theory- (Discretized Formulation)*

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Abstract

The scalar field is quantized in the discretized light-front framework following the *standard* Dirac procedure and its infinite volume limit taken. The background field and the nonzero mode variables do not commute for finite volume; they do so only in the continuum limit. A *non-local constraint* in the theory relating the two is shown to follow and we must deal with it along with the Hamiltonian. At the tree level the constraint leads to a description of the spontaneous symmetry breaking. The elimination of the constraint would lead to a highly involved light-front. Hamiltonian in contrast to the one found when we ignore altogether the background field. The renormalized constraint equation would also account for the instability of the symmetric phase for large enough couping constant.

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1. The possibility of building a Hamiltonian formulation of relativistic dynamics on light-front surface, $\tau = (t + z) = const.$, was pointed out by Dirac [1] and rediscovered by Weinberg [2] in the context of old-fashioned perturbation theory in the infinite-momentum frame. Since the longitudinal momentum k^+ (in the massive case) is necessarily positive and conserved, the vacuum structure looks simpler. The discretized light-cone quantized (DLCQ) field theory [3] in the context of perturbation theory does show simplifications and one hopes that the non-perturbative calculations may also be manageable through the numerical computation. The recent developments in the studies on Light-front Tamm-Dancoff Field Theory [4] to study non-perturbative effects (e.g., the relativistic bound states of light fermions) and the begining of a systematic study of perturbative renormalization theory [5] have the same motivation. The light-front approach may as described in ref. [4] throw some light on the relationship between the constituent quark picture and QCD with the dense sea of quarks and gluons in its vacuum.

The description of non-perturbative vacuum structure, for example, in the presence of a spontaneuos symmetry breaking scalar potential, Higgs mechanism, the fermionic condensates, and other related problems have, however, remained in the light-front framework without a clear understanding even at the tree level (see reviews in [3, 4]) in contrast to the description available in the case of equal-time quantization. We consider here, for concreteness and since it reveals most of the essential points, the light-front quantization of the massive scalar field in $1+1$ dimensions in detail. We will work for convenience in the discretized formulation and recover latter the physically significant predictions by taking the infinite volume limit to remove the spurious contributions introduced, for example, through the (zero modes) of finite volume delta and epsilon functions. Since the Lagrangian of any theory written in terms of light-front coordinates is necessarily singular (or degenerate), some new features emerge compared to the case of the equal-time quantization. For example, the background field variable ω and the field variable φ describing the fluctuations above the background may not be quantized independently and they satisfy a *nonlinear constraint* equation. Moreover their commutator is vanishing only in the continuum limit; in the discretized formulation with finite volume it does not. These features have been overlooked in DLCQ where the background field is ignored altogether and we work with a very simple Hamiltonian, simple commutation relations among the creation and destruction operators, and a trivial vacuum. This is *not* the case if non-perturbative ($\omega \neq 0$) effects are taken into account. We show below that in the light-front quantized theory we must now deal not only with the Hamiltonian operator but also with a non-linear operator constraint. It is not practical to solve the latter since it will lead to a very involved Hamiltonian unlike the simple one usually adopted in the literature [3] ignoring the background field. It follows from the discussion below that

the constraints are very useful ingredients and evidently may not be ignored in the light-front quantized field theory when considering non-perturbative effects. In the case under discussion it allows us to obtain the usual criterian for the spontaneous symmetry breaking at the tree level; there is no physical argument to minimize the light-front energy in contrast to the equal-time case. The renormalization may also be handle and the renormalized constraint equation may be used to obtain the quantum corrections in the value of the background field. It is worth remarking that this may be achieved in a straightforward fashion only in the continuum limit where ω and φ now commute. We comment also on the description of the well known phase transition for large coupling constants [6] and high order quantum correction to the well known "duality" [7] relation.

The Hamiltonian formulation for singular Lagrangians may be obtained using the Dirac method [8] which also allows us to find the constraints and the modified (Poisson) brackets necessary to take care of them among the surviving dynamical variables. Some constraints may of course be read off even at the Lagrangian level, say, by integrating the equations of motion but to quantize the theory we need to build a canonical framework. In the present context the Dirac method was attempted in the refs. [9, 10] in incomplete fashion with inconclusive results. The *standard* Dirac method requires that *all* the constraints (except the gauge-fixing ones) are derived from within the given Lagrangian. In ref. [9] the constraint $p \approx 0$ discussed below was missed while in ref. [10] it was proposed to modify the procedure itself and constrainsts added to the theory from outside. The authors also do not take the continuum limit properly, suggest modifications to the well known light-front commutators, do not consider the full implications of the constraint arising in the original Dirac procedure. There is also some misunderstanding about the term "zero mode". We have as the candidates the background field variable ω (e.g., the vacuum expectation value of the scalar field ϕ) and the variable $\widetilde{\varphi}\ (0)$ – the Fourier coefficient (or transform) for $k^+ = 0$ of the field φ. The kinematical constraint in the light-front frame work for massive theories requires that we must set $\widetilde{\varphi}\ (0)$ to be vanishing since, it corresponds to a *space-like* momentum vector k^μ in contrast to the nonzero (longitudinal) modes which correspond to the *time-like* vectors. This conclusion is also supported from axiomatic field theory considerations [11]. The ref. [9] does not even mention the background field ω which in the well known equal-time case characterizes the non-perturbative vacua when dealing with the spontaneously bronken symmetry potential while $\widetilde{\varphi}\ (0)$ is taken to be nonvanishing. Once clarified on these points and all the constraints derived from inside the theory, it is shown here that there is no need to modify the Dirac method thereby casting doubts on the validity of the Dirac method in general. It becomes clear (Sec. 2) also that there is no problem as regards to taking the infinite volume limit in straightforward way of the discretized formulation. In our context the constraints were not noted in still earlier work

[12], where only the light-front commutation relations were derived using the Schwinger's variational principle. We show here that they also emerge from the Dirac method and are accompanied by the (Hamiltonian and) nonlinear constraints. Similar constraints in the renormalized QCD in the light-front framework through a perturbative expansion may give useful hints to find new counter terms.

2. The light-front Lagrangian for the scalar field ϕ is

$$\int_{-\infty}^{\infty} dx[\dot{\phi}\phi' - V(\phi)], \tag{1}$$

where $V(\phi) \geq 0$, for example, $V(\phi) = (\lambda/4)(\phi^2 - m^2/\lambda)^2$, the potential with the wrong sign for the mass term and $\lambda \geq 0,\ m \neq 0$. Here an overdot and a prime indicate the partial derivations with respect to the light-front coordinates $\tau \equiv x^+ = (x^0 + x^1)/\sqrt{2}$ and $x \equiv x^- = (x^0 - x^1)/\sqrt{2}$ respectively and $x^+ = x_-, x_+ = x^-$ while $d^2x = d\tau dx$. The Euler equation of motion, $\dot{\phi}' = (-1/2)V'(\phi)$, where a prime on V indicates the variational derivative with respect to ϕ, shows that classical solutions, for instance, ϕ=const., are possible to obtain. We separate from the classical field ϕ the variable $\omega = \omega(\tau)$ (background field) corresponding to the operator which gives the vacuum expectation value of the would be quantized field ϕ. The *generalized* function ϕ, is then expressed as $\phi(x,\tau) = \omega(\tau) + \varphi(x,\tau)$. Here φ is an ordinary absolutely integrable function of x such that its Fourier transform $\widetilde{\varphi}\ (k,\tau)$ (or series) exist together with the inverse transform. Since under these assumptions $\varphi \to 0$ for $|x| \to \infty$ the vacuum expectation value of the quantized field φ vanishes. The vacuum expectation values of the momentum space operators $\widetilde{\varphi}\ (k,\tau)$ then also vanish for all $k \equiv k^+$. The quantized theory vacuum will be defined such that the expectation value of these operators for k different from zero is vanishing and we will assume the absence of the zero mode in φ, e.g., $\widetilde{\varphi}\ (0) = 0$ so that the space integral of the φ vanishes. This follows from the kinematical constraint in massive theory which allows only $k > 0$ as pointed out in Sec. 1. It is also clear that the discretized version may also be constructed and its infinite volume limit taken without any problems. The final expressions in the alternative treatments do coincide in the continuum limit if we adopt the commonly used interpretation of the space integrals over $[-\infty,\infty]$ as integral over $[-L/2, L/2]$ when $L \to \infty$ (Cauchy principal value) and for the delta function with vanishing argument, $2\pi\delta(0) = \lim_{L\to\infty}\int_{-L/2}^{L/2} dx$, where L is to be identified with the finite size extension in the x direction in the discretized formulation.

The discretized formulation is obtained through the following Fourier

series expansion

$$\phi(\tau,x)=\frac{q_0}{\sqrt{L}}+\frac{1}{\sqrt{L}}\sum_n{}' \, q_n(\tau)e^{-ik_nx}\equiv\frac{q_0}{\sqrt{L}}+\varphi(\tau,x) \tag{2}$$

where periodic boundry conditions are assumed, $\Delta=(2\pi/L), k_n=n\Delta$, $n=0,\ n=0,\pm1,\pm2,\cdots,\ \sum'_n$ indicates the summation excluding $n=0$. It is only *for convenience* that we have introduce an explicit factor $1/\sqrt{L}$ in the first term of (2); its limit in the continuum is simply $\omega(\tau)$. The discretized Lagrangian obtained by integrating the Lagrangian density in (1) over the finite interval $-L/2\le x\le L/2$ is given by

$$i\sum_n\ k_nq_{-n}\,\dot{q}_n\ -\ \int_{-L/2}^{L/2}dx\ V(\phi) \tag{3}$$

The momenta conjugate to q_n are $p_n=ik_nq_{-n}$ and the canonical Hamiltonian is found to be

$$H_c=\int_{-L/2}^{L/2}dx\ V(\phi) \tag{4}$$

The primary constraints are thus $p_0\approx0$ and $\Phi_n\equiv p_n-ik_nq_{-n}\approx0$ for $n\neq0$. We postulate initially the standard Poisson brackets at equal $\tau, viz, \{p_m,q_n\}=-\delta_{mn}$ and define the preliminary Hamiltonian

$$H'=H_c+\sum_n{}'\,u_n\Phi_n+u_0p_0 \tag{5}$$

On requiring the persistency in τ of these constraints we find the following weak equality [7] relations

$$\begin{aligned}\dot{p}_0 &= \{p_0,H'\}=\{p_0,H_c\}=\frac{1}{\sqrt{L}}\int_{-L/2}^{L/2}dxV'(\phi)\equiv-\frac{1}{\sqrt{L}}\,\beta(\tau)\approx0\ , &(6)\\ \dot{\Phi}_n &= \{\Phi_n,H'\}=-2i\sum_n{}'\,k_n\,u_{-n}\ -\frac{1}{\sqrt{L}}\int_{-L/2}^{L/2}dx\ V'(\phi)e^{-ik_nx}\approx0. &(7)\end{aligned}$$

From (6) we obtain an interaction dependent secondary constraint $\beta\approx0$ while (7) is a consistency requirement for determining $u_n,\ n\neq0$. Next we extend the Hamiltonian to

$$H''=H'+\nu(\tau)\beta(\tau)\ , \tag{8}$$

and check again the persistency of all the constraints encountered above making use of H''. We check that no more secondary constraints are generated if we set $\nu\approx0$ and we are left only with consistency requirements for determing the multipliers $u_n,\ u_0$. We easily verify that all the constraints $p_0\approx0,\ \beta\approx0$, and $\Phi_n\approx0$ for $n\neq0$ in our system are second class [7]. They may be implemented in the theory by defining Dirac brackets and this may be performed iteratively. We find $(n,m\neq0)$

$$\{\Phi_n,p_0\}=0 \qquad\qquad \{\Phi_n,\Phi_m\}=-2ik_n\delta_{m+n,0}\ , \tag{9}$$

$$\{\Phi_n, \beta\} = \{p_n, \beta\} = -\frac{1}{\sqrt{L}} \int_{-L/2}^{L/2} dx \left[V''(\phi) - V''\left(\frac{q_0}{\sqrt{L}}\right)\right] e^{-ik_n x} \equiv -\frac{\alpha_n}{\sqrt{L}} \ , \tag{10}$$

$$\{p_0, \beta\} = -\frac{1}{\sqrt{L}} \int_{-L/2}^{L/2} dx \ V''(\phi) \equiv -\frac{\alpha}{\sqrt{L}} \ , \tag{11}$$

$$\{p_0, p_0\} = \{\beta, \beta\} = 0 \tag{12}$$

We implement first the pair of constraints $p_0 \approx 0, \ \beta \approx 0$. The Dirac bracket $\{\}^\star$ with respect to them is easily constructed

$$\{f, g\}^\star = \{f, g\} - [\{f, p_0\}\{\beta, g\} - (p_0 \leftrightarrow \beta)] \frac{\alpha}{\sqrt{L}}^{-1}. \tag{13}$$

We may then set $p_0 = 0$ and $\beta = 0$ as strong relations since, for example, $\{f, p_0\}^* = \{f, \beta\} = 0$ for any arbitrary functional f of our canonical variables. The variable p_0 is thus removed from the theory. We conclude easily by inspection that the brackets $\{\}^*$ of the surviving canonical variables coincide with the standard Poisson brackets except for the ones involving q_0 and $p_n (n \neq 0)$

$$\{q_0, p_n\}^* = \{q_0, \Phi_n\}^* = -(\alpha^{-1}\alpha_n) \tag{14}$$

For the potential given just after eq. (1) above we find

$$\{q_0, p_n\}^* = \{q_0, \Phi_n\}^* = -\frac{3\lambda[2q_0 q_{-n} + \int_{-L/2}^{L/2} dx \varphi^2 \ e^{-ik_n x}]}{[3\lambda(q_0/\sqrt{L})^2 - m^2]L + 3\lambda \int_{-L/2}^{L/2} dx \varphi^2} \tag{15}$$

Next we implement the remaining constraints $\Phi_n \approx 0 \ (n \neq 0)$. We find

$$C_{nm} = \{\Phi_n, \Phi_m\}^* = -2ik_n \delta_{n+m,0} \tag{16}$$

and its inverse is given by ${C^{-1}}_{nm} = (1/2ik_n)\delta_{n+m,0}$. The *final* Dirac bracket which takes care of all the constraints of the theory is then given by

$$\{f, g\}_D = \{f, g\}^* - \sum_n{}' \frac{1}{2ik_n} \{f, \Phi_n\}^* \{\Phi_{-n}, g\}^*. \tag{17}$$

Inside this final bracket all the constraints may be treated as strong relations and we may now in addition write $p_n = ik_n q_{-n}$. It is straightforward to show that

$$\{q_0, q_0\}_D = 0 \quad \{q_0, p_n\}_D = \{q_0, ik_n q_{-n}\}_D = \frac{1}{2}\{q_0, p_n\}^*, \quad \{q_n, p_m\}_D = \frac{1}{2}\delta_{nm}. \tag{18}$$

It is also convenient to introduce the field $\pi(\tau, x)$

$$\pi(\tau, x) \equiv \varphi'(x) = \sum_n{}' \frac{p_n}{\sqrt{L}} e^{ik_n x} \tag{19}$$

which like φ is summed over the nonzero modes.

In order to remove the spurious finite volume effects in discretized formulation we must take the *continuum limit* $L \to \infty$. We have as usual: $\Delta = 2(\pi/L) \to dk$, $k_n = n\Delta \to k$, $\sqrt{L}q_{-n} \to \lim_{L\to\infty} \int_{-L/2}^{L/2} dx\varphi(x)e^{ik_nx} \equiv \int_{-\infty}^{\infty} dx\varphi(x)e^{ikx} = \sqrt{2\pi}\ \tilde{\varphi}\ (k)$ for $n, k \neq 0$. The sum over the nonzero mode in (3) goes over to $\sqrt{2\pi}\varphi(x) = \int_{-\infty}^{\infty} dk\ \tilde{\varphi}\ (k)e^{-ikx}$ along with the restriction $\int_{-\infty}^{\infty} dx\varphi(x) = \sqrt{2\pi}\varphi(0) = 0$. Since the zero mode gives rise to the vacuum expectation value of the quantized field ϕ it is clear that $(q_0/\sqrt{L}) \to \omega(\tau)$. From $\{\sqrt{L}q_m, \sqrt{L}q_{-n}\}_D = L\delta_{nm}/(2ik_n)$ following from the Dirac bracket between q_m and p_n for $n, m \neq 0$ in (19) we derive, on using $L\delta_{nm} \to \lim_{L\to\infty} \int_{-L/2}^{L/2} dxe^{i(k_n-k_m)x} = \int_{-\infty}^{\infty} dxe^{i(k-k')x} = 2\pi\delta(k-k')$, that

$$\{\tilde{\varphi}\ (k), \tilde{\varphi}\ (-k')\}_D = \frac{1}{2ik}\delta(k-k') \tag{20}$$

where $k, k' \neq 0$. On making use of the integral representation of the sgn function, $\in (x) = (i/\pi)\mathcal{P}\int_{-\infty}^{\infty}(dk/k)e^{-ikx}$ we are led to the light-front Dirac brackets for the field φ

$$\{\varphi(x), \varphi(y)\}_D = -\frac{1}{4} \in (x-y) \tag{21}$$

From (15) and (18) we derive similarly $\{\omega, \omega\}_D = 0$ and

$$\{\omega, \varphi(x)\}_D = -\left(\frac{3\lambda}{4}\right) lim_{L\to\infty} \frac{\int_{-\infty}^{\infty} dy \in (x-y)[2\omega\varphi(y) + \varphi(y)^2]}{(3\lambda\omega^2 - m^2)L + 3\lambda\int_{-\infty}^{\infty} dx\varphi(x)^2} \tag{22}$$

We find that at the classical level $\{\omega, \varphi(x)\}_D = 0$ only in the continuum limit and if the values of ω are such that the coefficient of L in the denominator of (22), e.g. $V''(\omega)$, is nonvanishing.

The resulting Dirac bracket (21) of φ is the usual light-front [12]. However, in the interacting theory we must take into account also of the implications of the *constraint* $\beta = 0$. Its explicit form for the potential under consideration is

$$L\left(\frac{q_0}{\sqrt{L}}\right)\left[\lambda\left(\frac{q_0}{\sqrt{L}}\right)^2 - m^2\right] + \lambda\int_{-L/2}^{L/2} dx\left[3\left(\frac{q_0}{\sqrt{L}}\right)\varphi^2 + \varphi^3\right] = 0 \tag{23}$$

which goes over to

$$\omega(\lambda\omega^2 - m^2) + \lambda\ lim_{L\to\infty}\frac{1}{L}\int_{-L/2}^{L/2} dx[3\omega\varphi^2 + \varphi^3] = 0. \tag{24}$$

At the tree level if ω is finite we find $V'(\omega) = \omega(\lambda\omega^2 - m^2) = 0$ which gives rise to $\omega = 0$ for the symmetric phase while $\omega = \pm(m/\sqrt{\lambda})$ for the asymmetric phases. For the free field theory or when we have the correct sign for the mass term (e.g., $m^2 \to -m^2$) in the interacting case we find $\omega = 0$. The

coefficient of L in the denominator of (22) is non-vanishing for these values for ω and consequently the right hand side of (22) vanishes for these cases on removing the finite size effects by taking the continuum limit. The *final* Hamiltonian coincides with H_c

$$H = H_c = \int_{-\infty}^{\infty} dx \left[\frac{1}{2}(3\lambda\omega^2 - m^2)\varphi^2 + \lambda(\omega\varphi^3 + \frac{1}{4}\varphi^4) + \frac{1}{4\lambda}(\lambda\omega^2 - m^2)^2 \right], \tag{25}$$

and the Lagrange equations of motion are recovered from the Hamiltons equations assuring the self-consistency [8] of the procedure. It is clear from (24) and (25) that the elimination of ω using the constraint would lead to a very involved Hamiltonian except in the case we ignore the background field altogether. It is clear also that all these results follow *immediately* if we had worked directly in the continuum and interpreted $\delta(0) = L/(2\pi)$ with $L \to \infty$.

3. The quantized theory is obtained by the correspondence $i\{f, g\}_D \to [f, g]$ where the quantities inside the commutator are the corresponding quantized operators. In the interacting theory the operator ω is seen to commute with itself and with the nonzero modes only in the continuum limit and we are left with the *nonlinear operator constraint* together with the Hamiltonian operator. The higher order corrections to (24) in the renormalized field theory will alter the tree level values of ω since, we do *not* have any physical considerations to normal order the constraint equation. The commutation relations of φ may be realized in momentum space through the expansion ($\tau = 0$)

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \frac{\theta(k)}{\sqrt{2k}} [a(k)e^{-ikx} + a^{\dagger}(k)e^{ikx}] \tag{26}$$

where $a(k)$ and $a^{\dagger}(k)$ satisfy the canonical commutation relations, viz, $[a(k), a(k')^{\dagger}] = \delta(k - k'), [a(k), a(k')] = 0$, and $[a(k)^{\dagger}, a(k')^{\dagger}] = 0$ while the ω commutes with them and thus is proportional to the identity operator in the Fock-space. The vacuum state is defined by $a(k)|vac\rangle = 0, k > 0$. The longitudinal momentum operator is $P^+ = \int dx : \varphi'^2 :$ and the light-front energy is $P^- = H = \int dx : V(\phi) :$ where we normal order with respect to the creation and destruction operators to drop unphysical infinities and we find $[a(k), P^+] = ka(k), [a^{\dagger}(k), P^+] = -ka^{\dagger}(k)$. The values of $\omega = \langle|\phi|\rangle_{vac}$ obtained from solving $V'(\omega) = 0$ *characterize* the (non-perturbative) vacua and the Fock space is built by applying the nonzero mode operators on the corresponding vacuum state. The reflection symmetry $\phi \to -\phi$ is broken spontaneously when $\omega \neq 0$. We remark that in view of the constraint (24) and the non-commutability of ω with the operators $a(k)$ and $a^{\dagger}(k)$ in finite volume a nonperturbative computation in the DLCQ would be much more difficult to handle than in the continuum formulation except for in the usually considered case where the background field is ignored altogether. The

high order quantum corrections would alter the value of ω as determined from the renormalized constraint equation and it may be shown that we do obtain significant deviations [13] from the well known "duality" relation [7]. It is worth remarking that in the present case the zero mode (of phi) is the background field and is essentially a c-number over the Fock space. This is in contrast to the case of the light-front quantization of the bosonic version of the Schwinger model obtained by functionally integrating over the fermions and introducing the scalar field to obtain a local field theory. Here in order to ensure at the quantum level the symmetry of the Lagrangian with respect to the shift by a constant of the scalar field (chiral symmetry), a zero mode from the only other field available, viz, the gauge field, must be an operator and canonically conjugate to the zero mode operator of the scalar field. This model discussed by modified procedure [14] can also be handled by following the standard Dirac procedure.

4. We conclude thus that in view of the constraint in the light-front quantized field theory we do obtain a description of the spontaneous symmetry breaking. The treatment of the non-perturbative effects in the DLCQ, contrary to the common understanding, seems to be quite a difficult task in view of the non-commutability of the background field with the nonzero modes and the presence of the non-linear constraint. The continuum formulation is more convinient in that it can be renormalized straightforwardly and high order corrections computed from the renormalized constraint equation.

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